

*Entry Task:* Plot these polar points

(1)  $(r, \theta) = (2, -\pi/4)$

(2)  $(r, \theta) = (1, 2\pi/3)$

(3)  $(r, \theta) = (-1, \pi/4)$

## 10.3 Polar Coordinates

### Polar

Given  $(r, \theta)$

1. Stand at origin facing the positive  $x$ -axis.

2. Rotate by  $\theta$ .

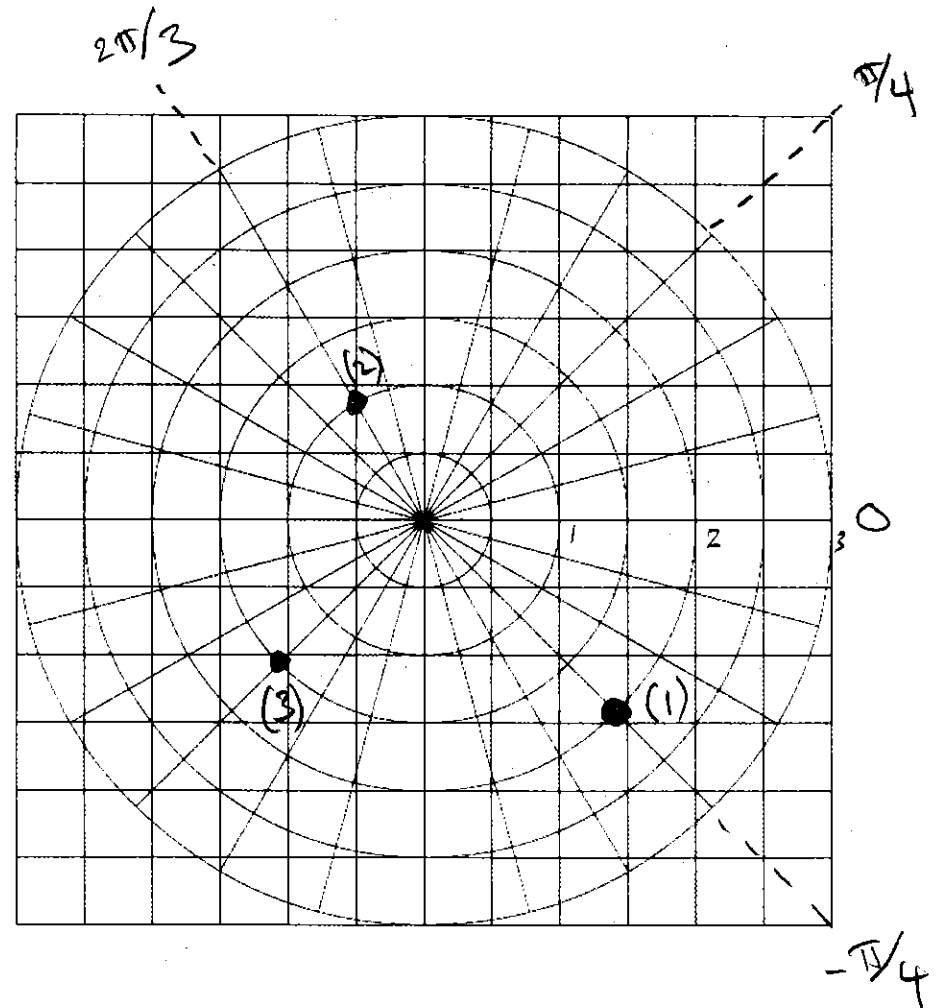
pos. = ccw,

neg. = clockwise

3. Walk  $r$ -units in direction you are facing.

pos. = forward

neg. = backward



From trig we already know:

$$x = r \cos(\theta), \quad y = r \sin(\theta)$$

$$\tan(\theta) = \frac{y}{x}, \quad x^2 + y^2 = r^2$$

Exercise:

1. Describe all pts where  $r = 3$ .

$$\Rightarrow x = 3 \cos \theta \quad \text{AND} \quad x^2 + y^2 = 9$$

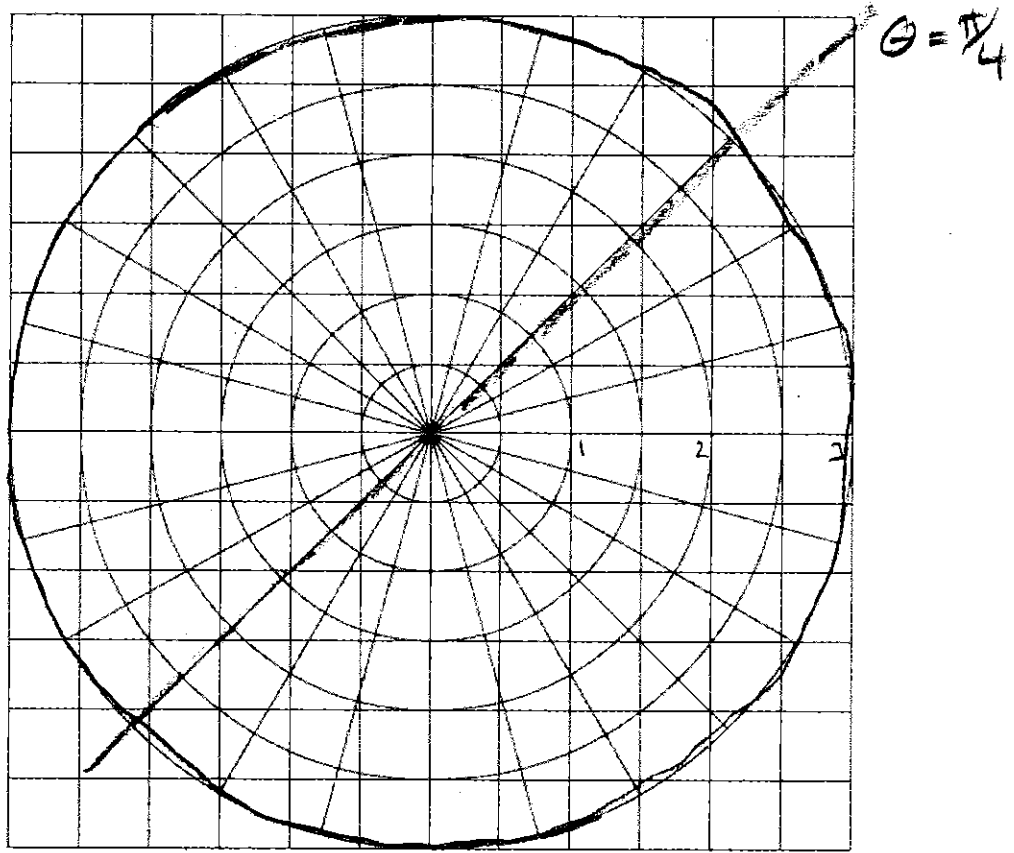
$$y = 3 \sin \theta \quad \text{CIRCLE!}$$

2. Describe all pts where  $\theta = \pi/4$ .

$$\theta = \pi/4, \quad r = \text{ANYTHING} \quad \leftarrow \text{SAME}$$

$$\tan(\pi/4) = \frac{y}{x} \quad \leftarrow \text{SAME}$$

$$\Rightarrow y = x$$



$$r = 3 \quad \theta = \text{ANYTHING} \quad \leftarrow \text{SAME}$$

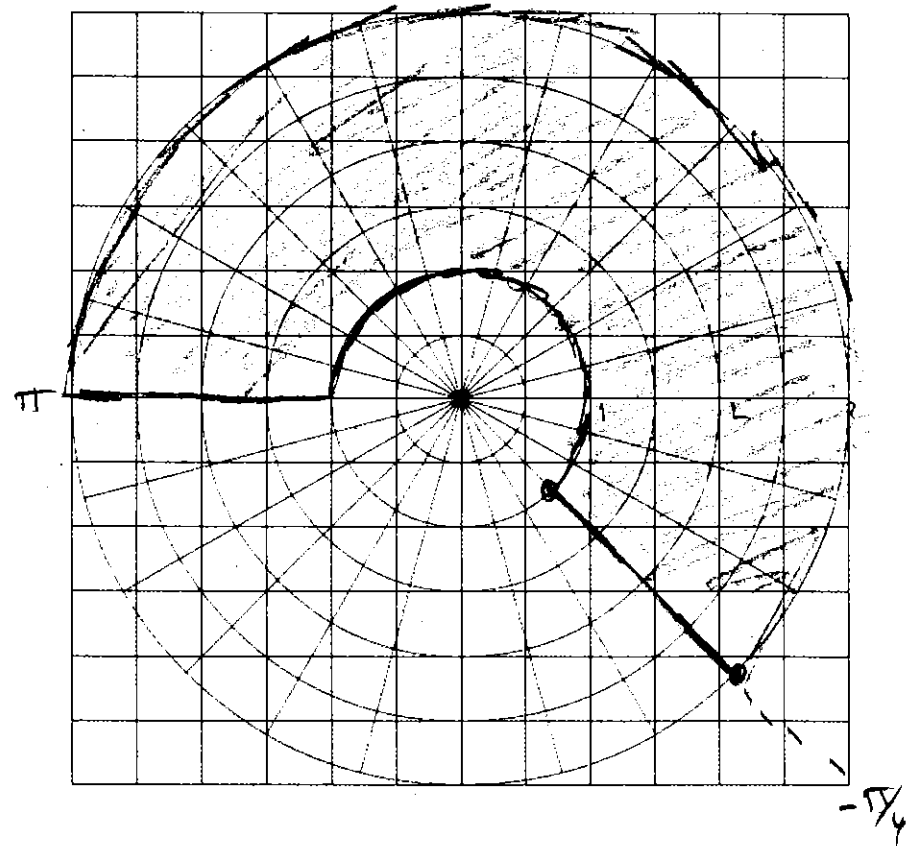
$$x^2 + y^2 = 9 \quad \leftarrow \text{SAME}$$

# Polar Regions

1. Describe all pts where

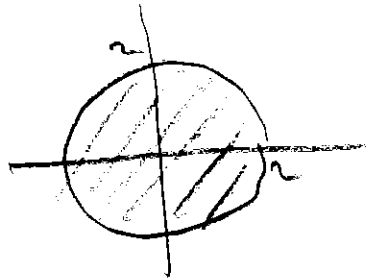
$$-\frac{\pi}{4} \leq \theta \leq \pi \text{ and } 1 \leq r \leq 3$$

↑ "INNER"      ↑ "OUTER"



2. Describe all pts where

$$0 \leq \theta \leq 2\pi \text{ and } 0 \leq r \leq 2$$



## Plotting Polar Curves

*Option 1:* Try to convert to  $x$  and  $y$ .  
Then hope you recognize the curve.

### *Option 2:* **Plot points!**

Start with  $0, \pi/2, \pi, 3\pi/2$  (intercepts).  
For more detail do multiples of  $\pi/6$   
and  $\pi/4$ .

Example: Graph  $r = \sin(\theta)$

$$r \sin \theta = y \Rightarrow \sin \theta = \frac{y}{r}$$

$$\Rightarrow r = \frac{y}{\sin \theta}$$

$$\Rightarrow r^2 = y$$

$$\Rightarrow x^2 + y^2 = y$$

$$\Rightarrow x^2 + y^2 - y = 0 \quad ??$$

COMPLETE SQUARE

$$x^2 + y^2 - y + \frac{1}{4} - \frac{1}{4} = 0$$

↳ HALF SQUARE

$$x^2 + (y - \frac{1}{2})^2 = \frac{1}{4}$$

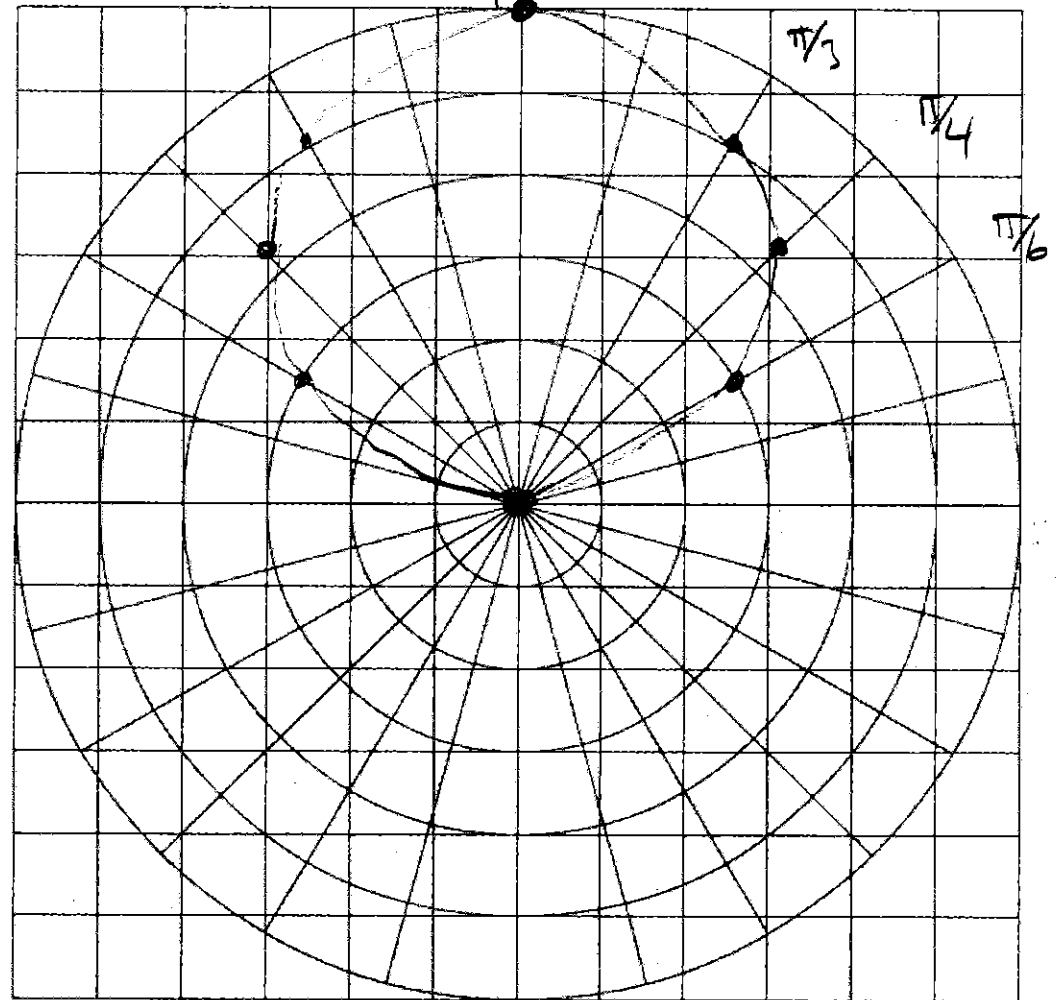
CENTER:  $(0, \frac{1}{2})$

RADIUS:  $\frac{1}{2}$

$\theta$	0	$\pi/2$	$\pi$	$3\pi/2$	$2\pi$
$r$	0	1	0	-1	0

$\theta$	$\pi/6$	$\pi/4$	$\pi/3$	$2\pi/3$	$3\pi/4$	$5\pi/6$
$r$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	

$\approx 0.7$     $\approx 0.86$     $\approx 0.86$

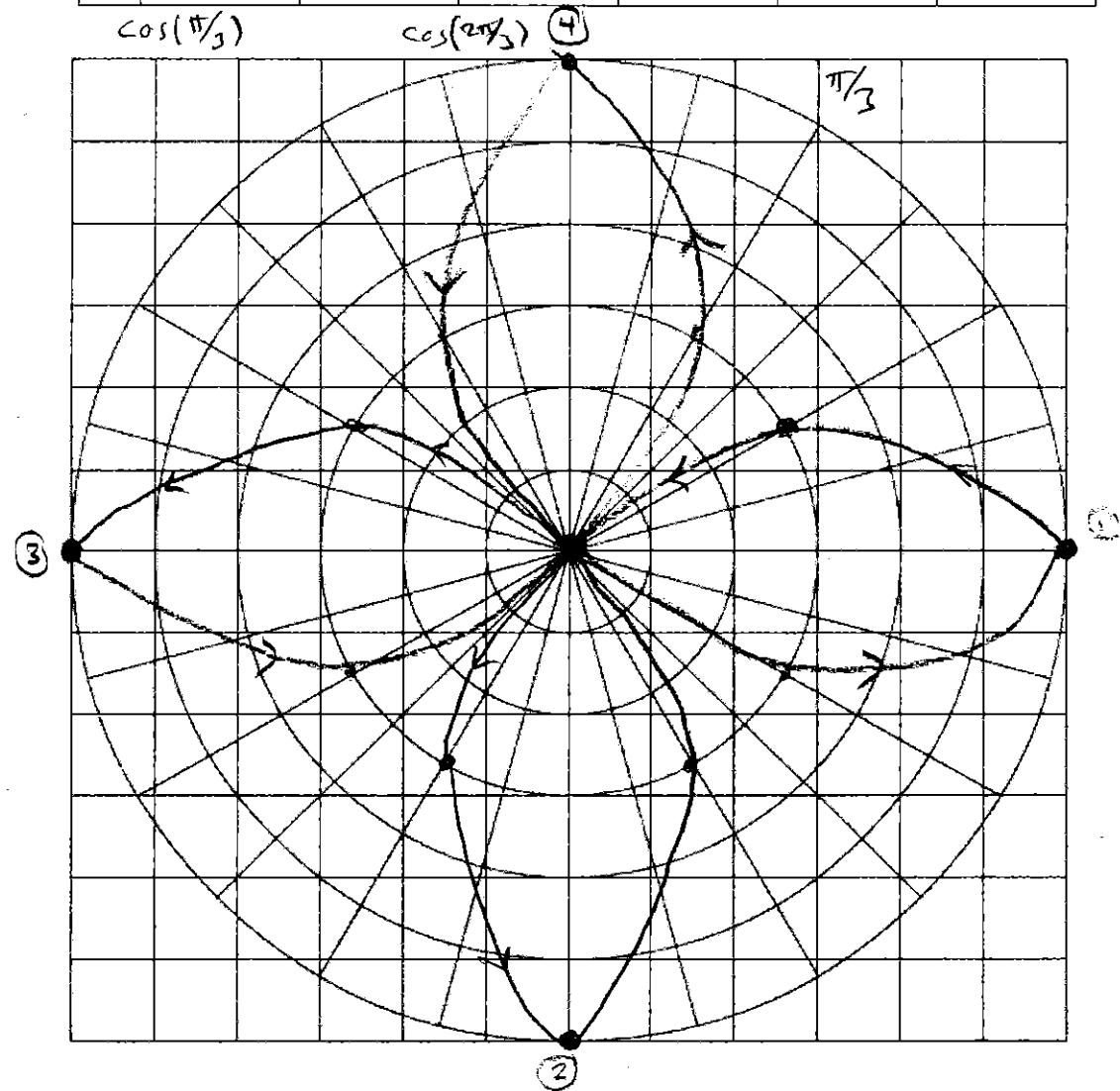


Example: Graph  $r = \cos(2\theta)$

$\theta$	0	$\pi/2$	$\pi$	$3\pi/2$	$2\pi$
$r$	1	-1	1	-1	1

COULD USE HALF-ANGLE  
 THEN TRY TO GET XY-EQUATIONS!  
 BUT IT WON'T BE NICE AND  
 YOU WON'T RECOGNIZE  
 THE XY-CURVE!  
 SO JUST PLOT PTS

$\theta$	$\pi/6$	$\pi/4$	$\pi/3$	$2\pi/3$	$3\pi/4$	$5\pi/6$
$r$	$1/2$	0	$-1/2$	$-1/2$	0	$1/2$



Q | DESCRIBE ONE LOOP

$$-\pi/4 \leq \theta \leq \pi/4$$

$$0 \leq r \leq \cos(2\theta)$$

An old exam question:

The four polar equations below each match up with one of the six pictures. Identify which match.

1.  $r = \sqrt{\theta}$  ← SPINAL (r getting bigger)
2.  $r = 1 - 2\cos(\theta)$
3.  $r = 1 + \sin(2\theta)$
4.  $r = 9\cos(\theta)$

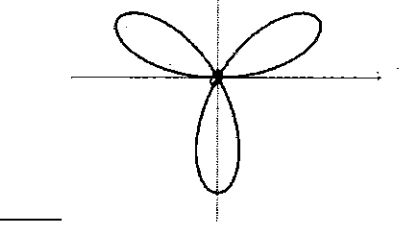
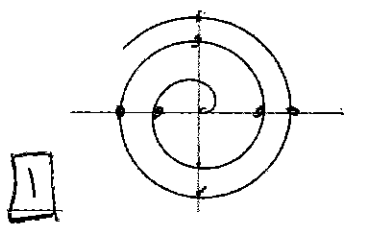
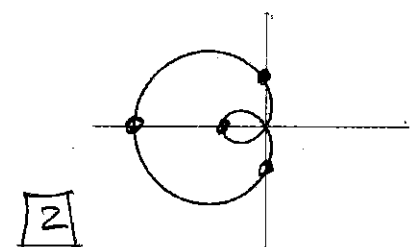
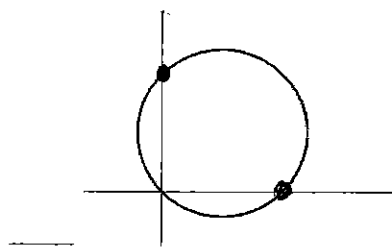
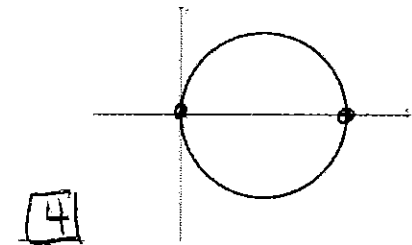
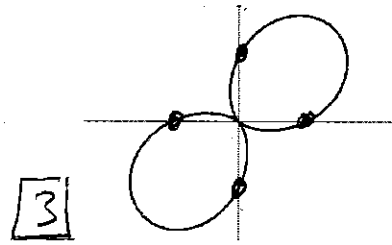
2

$\theta$	0	$\frac{\pi}{2}$	$\pi$	$\frac{3\pi}{2}$
	-1	1	3	1



3

$\theta$	0	$\frac{\pi}{2}$	$\pi$	$\frac{3\pi}{2}$
	1	1	1	1



4

$\theta$	0	$\frac{\pi}{2}$	$\pi$	$\frac{3\pi}{2}$
	9	0	-9	0

